ULTURB
Scientific Description

Background

To date, clear air turbulence (CAT) forecast techniques have been an amalgamation of mostly empirical rules and equations, most of which are bases on perceived connections between observed atmospheric patterns and aircraft turbulence reports. McCann (2001) demonstrated that these techniques look at the environmental setup for CAT as measured directly or indirectly by the Richardson number

$$Ri \equiv \frac{\frac{1}{\Theta}\frac{d\Theta}{dz}}{\left(\frac{d\mathbf{V}}{dz}\right)^2}$$

where Θ is the potential temperature and **V** is the wind velocity. The numerator is the layer's stability, and the denominator is the layer's wind shear.

Layers above the atmospheric boundary layer are rarely favorable for turbulence because it is necessary for $Ri < 0.25$ for turbulence to form. Assumed in these techniques is some undefined process that locally alters the environment so the atmosphere can become turbulent. The lower the environmental *Ri*, the higher probability of turbulence. Unfortunately, these techniques overforecast CAT because often low *Ri* environments are smooth. The situation is analogous to thunderstorm forecasting only considering conditional instability. Indeed, the thunderstorm probability increases with a lower Lifted Index, but successful thunderstorm forecasts include consideration of triggers. Similarly, CAT forecasting techniques that include a trigger analysis should reduce the uncertainty of environment-only techniques.

Gravity waves, which are ubiquitous in the atmosphere, alter both the environmental wind shear and the stability as they move though. McCann showed that, under the influence of a gravity wave, the local Richardson number (Ri_L):

$$Ri_L = Ri_E \frac{1+\hat{a}\cos\varphi}{\left(1+Ri_E{}^{1/2}\hat{a}\sin\varphi\right)^2}$$

where Ri_E is the environmental Richardson number, φ is the gravity wave phase angle, and $\hat{a}$ is the wave amplitude non-dimensionalized by multiplying the actual wave amplitude by the stability and dividing by the Doppler-adjusted wind speed, |**V-c**|, where **c** is the wave phase velocity. There are sufficient observations/numerical model forecasts of the wind and temperature to compute environmental stability, wind shear, and Ri_E. The gravity wave amplitude and phase velocity are unknowns that make computing Ri_L difficult. Obviously, research must focus on gravity wave characteristics in order to apply the theory with confidence.

However, assuming that $\hat{a}$ can be computed in some way, the formula shows the local increases or decreases in stability and wind shear within the gravity wave oscillation depending on the gravity wave phase angle, φ. Turbulence forecasters are only interested in the modifications that will produce Ri_L less than 0.25. When $\varphi = \pi$ and $\hat{a} > 1$, $Ri_L < 0.0$, a condition for wave breaking. When $\varphi = \pi/2$, the local wind shear is maximized, so when $Ri_E(2-\hat{a}) < 1$, then $Ri_L <$ 0.25. For $\hat{a} > 2$, Ri_L is always less than 0.25. Figure 1 depicts the graph of this curve. As $\hat{a} \rightarrow 2$, the upper limit on Ri_E for turbulence approaches infinity. Below the curve are Ri_E and $\hat{a}$ combinations that are turbulent.

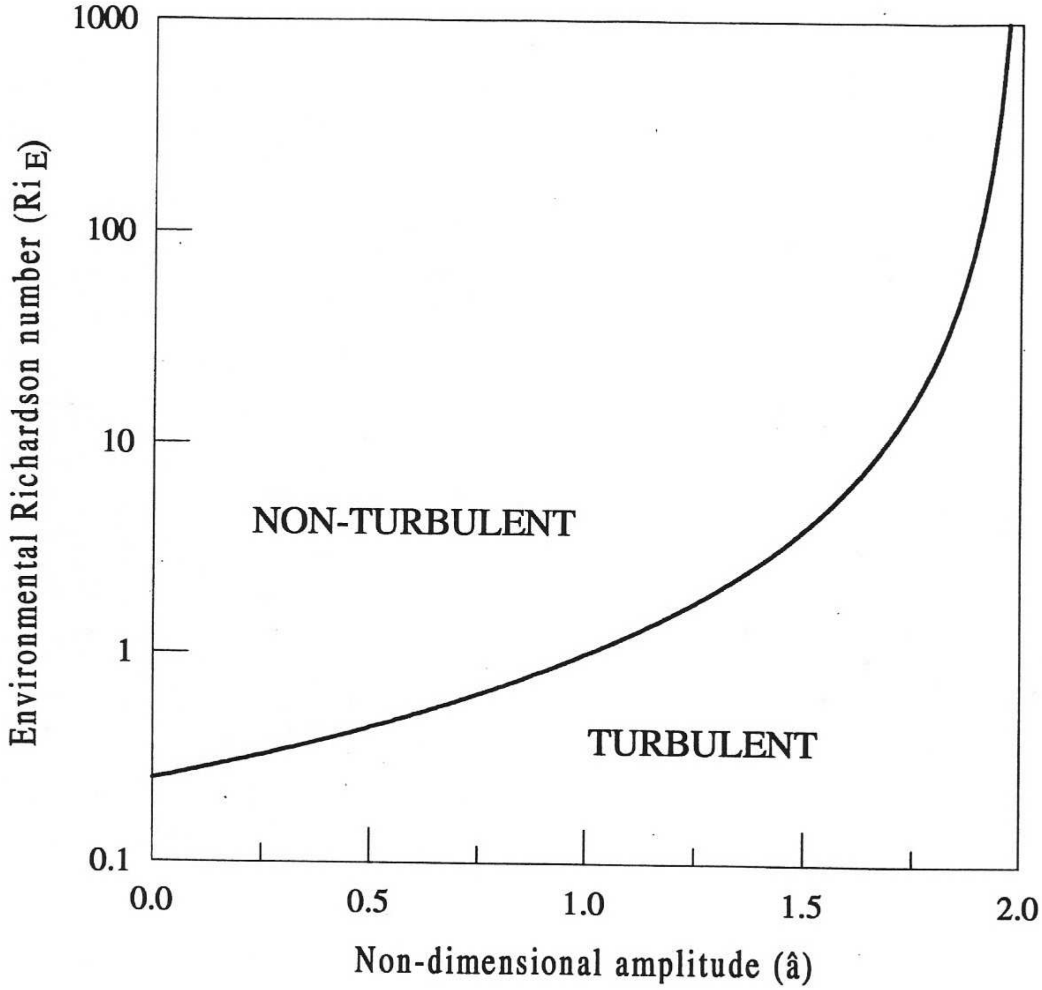

Figure 1. Curve of the bounding value of the environmental Richardson number as a function of the non-dimensional amplitude (â). When â falls in the TURBULENT region, a gravity wave will locally increase the wind shear sufficiently to reduce the local Richardson number to < 0.25.

This theory quantifies current environmental-only CAT forecast techniques. The lower the environmental Richardson number, the higher the probability that a gravity wave with sufficient non-dimensional amplitude will reduce the local Richardson number to less than 0.25.

If gravity waves trigger CAT in this manner, then improving CAT forecasting techniques requires future knowledge of the gravity waves that cause CAT. There are many sources of gravity waves, and any gravity wave may be a potential turbulence producer. Rossby (1938) first noted that whenever forces on air parcels are unbalanced, the resultant adjustment may excite gravity waves. There are several techniques for diagnosing unbalanced flow, and McCann (2004) found two that were well correlated with organized patterns of turbulence pilot reports.

Two unbalanced flow diagnostics

The Lagrangian time derivative of the continuity equation in isobaric coordinates is

$$\frac{d}{dt}\left(\frac{\partial \omega}{\partial p}\right) = \frac{d}{dt}\left(-\nabla_H \cdot \mathbf{V}\right)$$

where ω is the isobaric vertical velocity, and *p* is the pressure. The left side implies a change in the vertical profile of vertical velocity which implies a vertical acceleration. A vertical acceleration must be caused by a vertical force which, with positive stability, causes a gravity wave. Thus the total time derivative of divergence may be used as a proxy for the occurrence of gravity waves.

One can compute the total divergence tendency knowing the divergence at two times. However, the divergence tendency may be computed from variables at a single time. Taking the divergence of both sides of the equation of motion and after considerable algebra (Haltiner and Williams 1980) the total divergence equation becomes

$$\frac{\partial D}{\partial t} + \mathbf{V} \cdot \nabla D + \omega \frac{\partial D}{\partial p} = \underbrace{-D^2}_{A} \underbrace{- \frac{\partial \mathbf{V}}{\partial p} \cdot \nabla \omega}_{B} + \underbrace{2\mathrm{J}(u,v)}_{C} \underbrace{- \beta u}_{D} + \underbrace{f\zeta}_{E} \underbrace{- \nabla^2 \Phi}_{F} + \underbrace{\nabla \cdot \mathbf{F}}_{G}$$

where *D* is the divergence, *u*,*v*, and ω are the three components of the total wind velocity, J is the Jacobian operator, *f* is the Coriolis parameter, β is ∂*f*/∂*y*, ζ is the relative vorticity, Φ is the geopotential, and **F** is the frictional force. The left hand side are the three Eulerian components of the Lagrangian time derivative of divergence. On the right side, term A is the effect divergence has on its tendency, term B is the vertical wind shear/horizontal gradient of vertical motion interaction, term C accounts for the horizontal wind shear, and term D is the effect due to convergence of longitude at the poles. Terms E and F are terms related to the relative vorticity and the geostrophic vorticity, respectively. Term G contains the subgrid processes that change divergence including the turbulence itself. Unfortunately, term G cannot be evaluated like the other terms. Term D is two orders of magnitude smaller than the other terms and can safely be ignored. All other terms can be significant contributors to divergence tendency.

With the hypothesis that total divergence tendency causes gravity waves in a stable environment, it is further hypothesized that the magnitude of the total divergence tendency is proportional to and may be used to estimate $\hat{a}$. Furthermore, knowing $\hat{a}$, the maximum production of turbulent kinetic energy (TKE_L) from gravity wave enhanced wind shear is (McCann 2001):

$$TKE_L = K_m \left(\frac{\partial \mathbf{V}}{\partial z} \right)^2 (1 + Ri_E^{1/2} \hat{a})^2 \qquad \hat{a} > \left(2 - \frac{1}{Ri_E} \right)$$

where K_m is the momentum eddy diffusivity or eddy viscosity.

McCann (2004) showed that, of the individual terms in the divergence tendency equation, a negative term C showed the best positive correlation with pilot reports in strong, organized turbulence outbreak cases, and a positive term E was next best. The results for term B also showed good correlation. This suggested two ways to estimate $\hat{a}$, one as the negative of terms B + C and the other as the positive term E.

The ULTURB algorithm

Because of the limited knowledge of gravity wave amplitude and phase velocity, ULTURB assumes that the magnitude of the two divergence tendency diagnostics in a layer is proportional to the non-dimensional triggering gravity wave amplitude ($\hat{a}$) in that layer. This simple assumption ignores the wave elements that are important for a complete trigger analysis, but until research provides theoretical and/or observational details of wave structure, this assumption will have to suffice.

ULTURB treats the two unbalanced flow diagnostics as independent. Each is divided by its own proportionality constant to compute $\hat{a}$. The proportionality constants ($C_{-(B+C)} = -0.5 \times 10^{-8}$ sec^{-2} and $C_E = 1.2 \times 10^{-8}$ sec^{-2}) were determined from examining TKE_L production with a sample of known severe CAT outbreaks.

At each model grid point and in each layer, ULTURB first computes each of the two divergence tendency trigger terms. The non-dimensional amplitude is the maximum $\hat{a}$ of the two trigger terms. Then ULTURB determines the ($\hat{a}$,Ri) combination. If this combination is above the Fig. 1 curve, then $TKE_L = 0$. If it is below the Fig. 1 curve, then TKE_L is that determined from the equation above ($K_m = .054$, from McCann [1999]). ULTURB limits the non-dimensional amplitude, $\hat{a} = 2.5$ to account for nonlinear limitations on wave amplification.

Operational interpretation

The following figures show a severe turbulence outbreak case on 13 November 2002. Figures 2-4 show the various pertinent divergence tendency terms, and Figure 5 shows the resultant ULTURB computation of TKE dissipation and the pilot reports received.

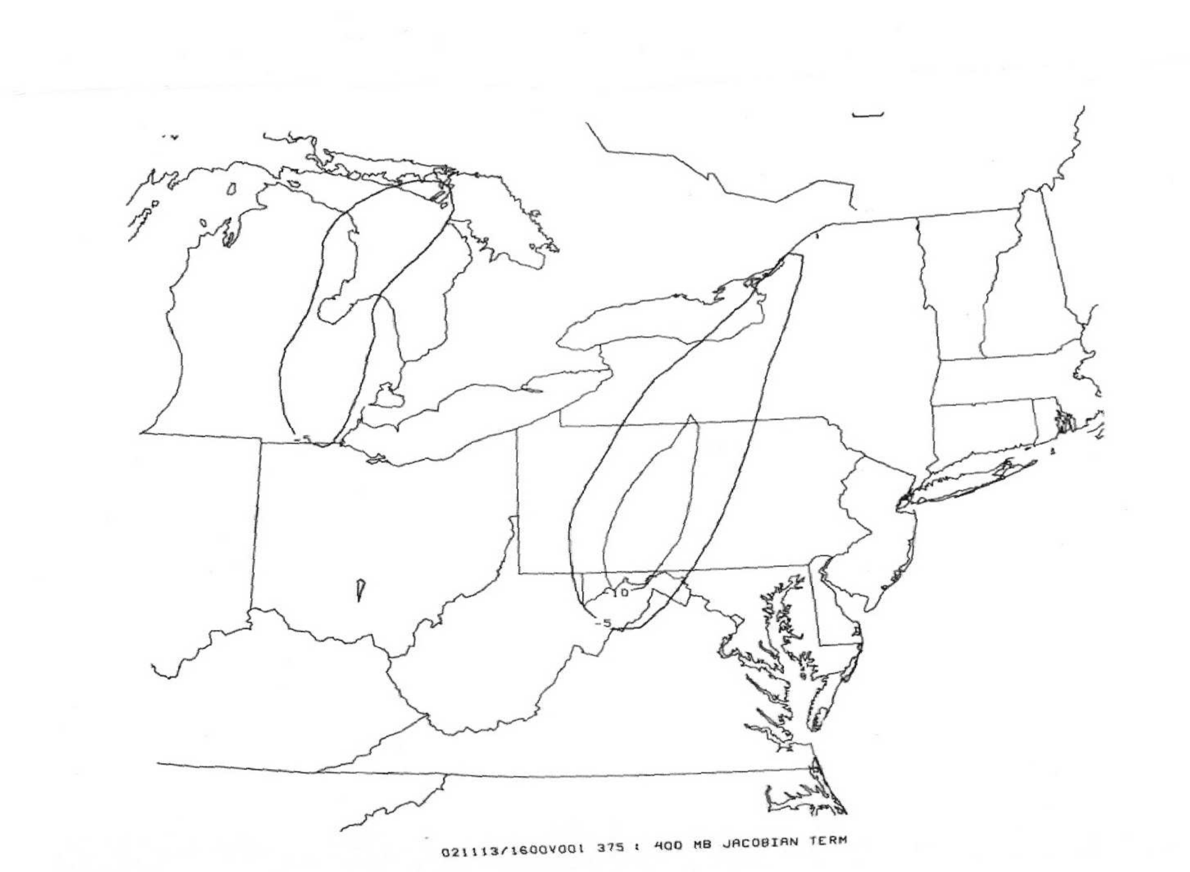

Figure 2. The divergence tendency term C as computed from the 1-hour RUC2 1500 UTC 13 Nov 2002 forecast.

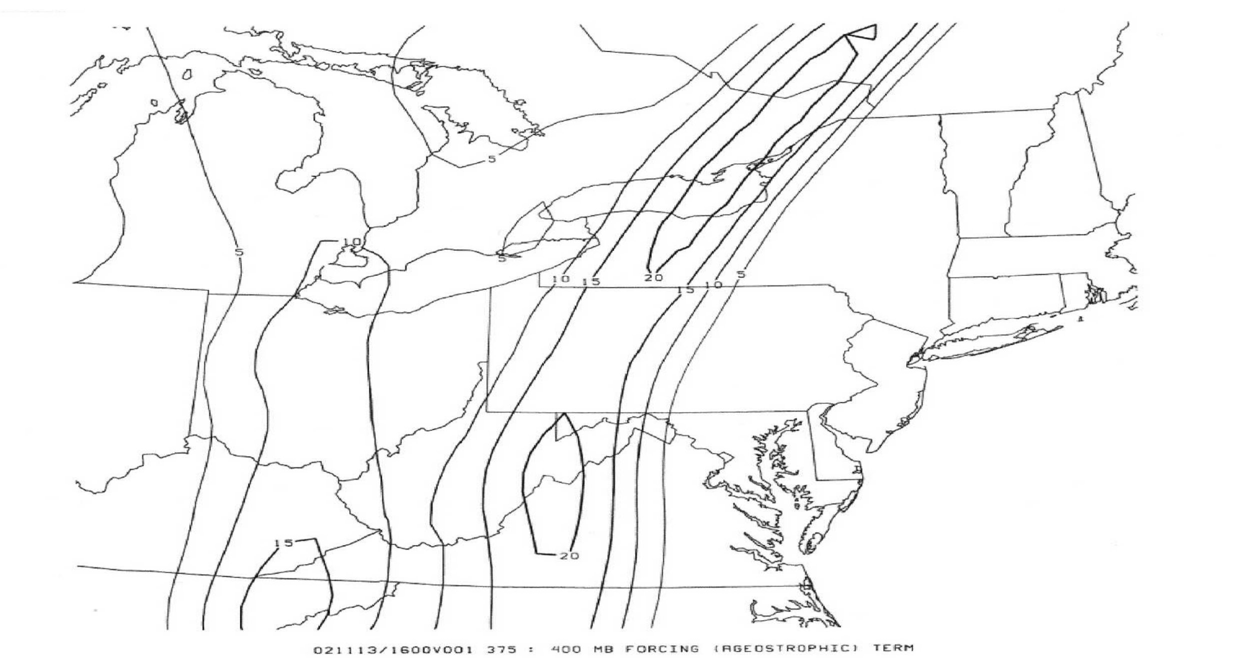

Figure 3. Same as Fig. 2 except the divergence tendency term E.

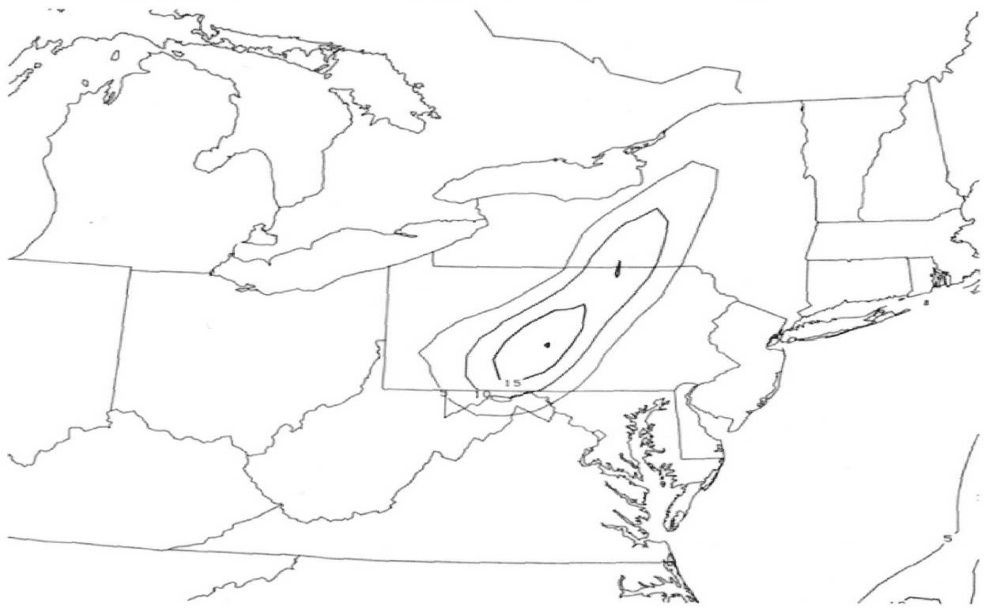

Figure 4. Same as Fig. 2 except divergence tendency term B.

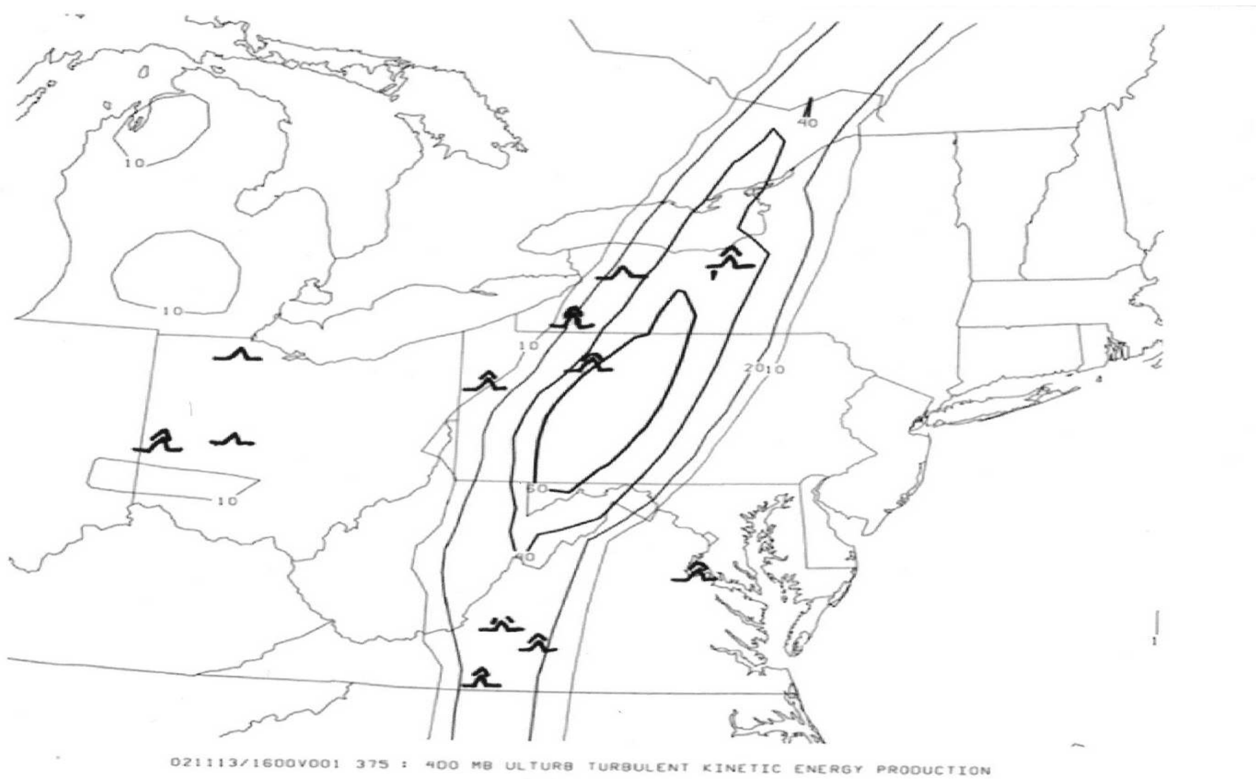

Figure 5. ULTURB TKE production (10^{-3} j s^{-1}) computed from the 1-hour RUC2 1500 UTC 13 Nov 2002 forecast for the layer near FL250. Also plotted are the pilot reports between FL220 and FL280 between 1500 UTC and 1700 UTC

The eddy K-values in ULTURB largely determine the magnitude of the TKE dissipation. Because they were designed similarly to those in the boundary layer (McCann 1999), the threshold values for choosing between LIGHT, MODERATE, and SEVERE turbulence should be similar. The table below repeats those thresholds.

Intensity	Value
Smooth/Light	.000
Light/Moderate	.016
Moderate/Severe	.035

While ULTURB computes TKE dissipation in all atmospheric layers, it was designed and tested for turbulence at high performance aircraft cruising levels. It has not been tested in the lower atmosphere and is not displayed for layers below 700 mb.

References

McCann, D.W., 1999: A simple turbulent kinetic energy equation and aircraft boundary layer turbulence. *Natl. Wea. Digest*, **23(1,2)**, 13-19.

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